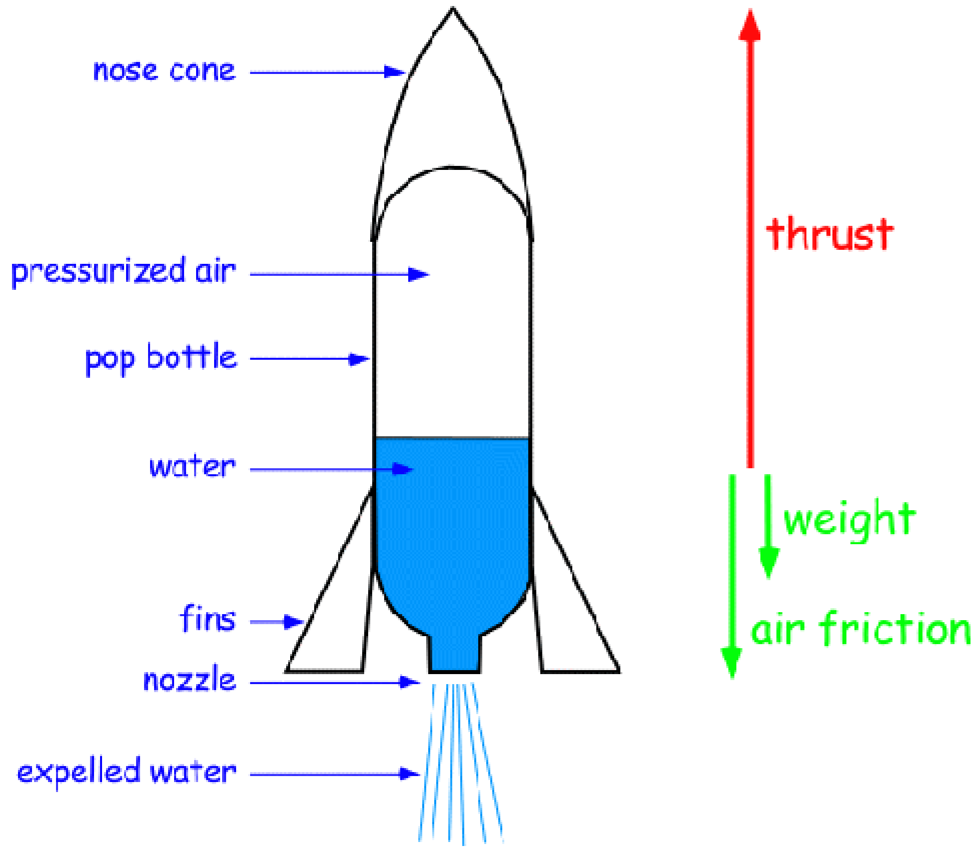
It's not Rocket Science.... Oh Wait!!!

Anatomy of a Bottle Rocket



Conservation of Momentum

Given two objects in an isolated system, the momentum lost by one object is equal to the momentum gained by the other object. – The Law of Conservation of Momentum

This tells us that the momentum gained by the rocket is equal to the mass of the fuel expelled (i.e., the water) multiplied by the exit velocity of the fuel leaving the rocket through the nozzle.

Initial Velocity

After all of the fuel has been expelled, the total change in momentum, or the **impulse**, of the rocket body can be used to find the rockets velocity.

All of the fuel will leave the rocket very quickly, so it is reasonable to assume that the velocity of the rocket, after **all** of the fuel has been expelled, is the initial velocity.

Objective

You must design your rocket to strike a target be deciding these values:

- Air Pressure
- Water Volume
- Pitch Angle

There is more than one correct solution but some factors may affect your accuracy.

Hint: Things like air drag and the time the water takes to leave the rocket are difficult to calculate. Try to design your rocket so that these sources of error will matter less.

Finding the Initial Velocity

Determining the exact trajectory of the rocket requires some complicated math, so we will using a computer to help. Computer aided design is often used in the aerospace industry. The simulator is available online at:

http://www.sciencebits.com/RocketCalculator

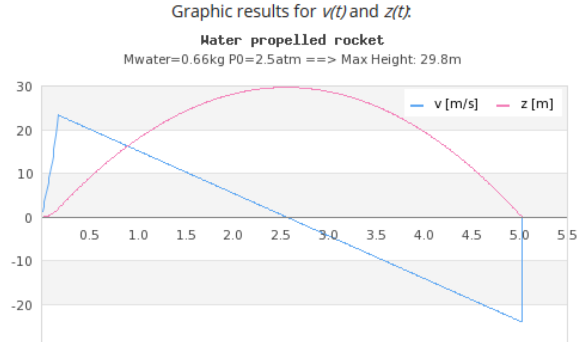
This simulator assumes the rocket is being shot straight up (i.e., with a pitch of 90 degrees), but your rocket will be launched at an angle. But the maximum velocity will be the same regardless of the angle.

Enter the water rocket parameters, and press "OK, launch!" to simulate it.

Empty rocket weight (i.e., without water)	M_0	0.10	Kg
Initial amout of water	M_w	0.66	Kg
Bottle Volume	V_b	2.00	liter
Initial pressure	P_0	2.50	atm
Gravitational Constant	g	9.80	m/s^2
Drag Coefficient (~0.5 for smooth cylinder)	C_d	0.00	
Simulation time step	Δt	0.010	s
Bottle radius	R_z	5.00	cm
Nozzle radius	R_n	1.00	cm

OK, launch!

Graphic results for *v(t)* and *z(t)*:



You could try to estimate the maximum velocity by looking at the graph that is generated, or you could use the Max Height and some basic kinematics equations to get a more precise value (make sure to set the Drag Coefficient C_d to zero if you use this method).

Consider this question: If a ball with mass M_0 is thrown straight upwards on Earth where the acceleration due to gravity is g, and reaches a maximum height H, what is the initial velocity of the ball?

Answer: $v_i = \sqrt{2gH}$

Striking the Target

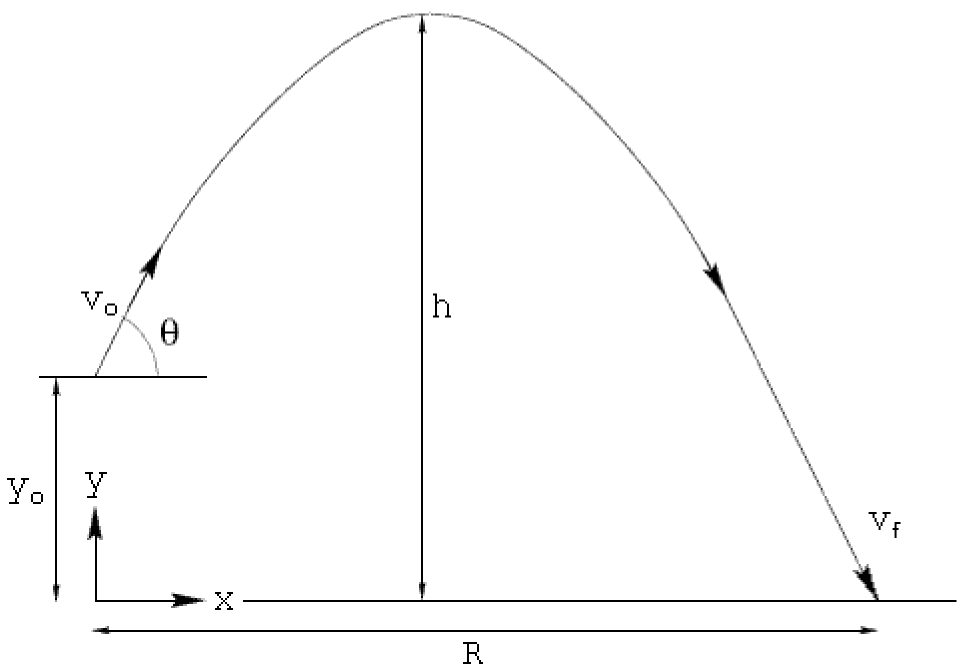
Once you have found the initial velocity, we can estimate the distance the rocket will travel be assuming no air resistance and using the following formulae:

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{dg}{v_i^2}\right) \qquad\qquad d = \frac{v_i^2}{g}\sin(2\theta)$$

Where v_i is the initial or maximum velocity, g is the acceleration due to gravity on Earth, and θ is the pitch angle at which the rocket is launched.

Bonus: Improving the Initial Velocity Approximation

So far, we have assumed that the maximum velocity and the initial velocity are the same. This approximation works well, but in reality, the rocket is already a few meters above the ground when the maximum velocity is achieved. We can improve our design by adding an initial height y_0,



The parabolic trajectory of a projectile

First, find the time t when the rocket will hit the ground by setting $y = 0$ and solving for t,

$$y = y_0 + v_0 t \sin(\theta) - 0.5 g t^2$$

Now, find the distance $R = x(t)$ the rocket has travelled when it strikes the ground,

$$x(t) = x_0 + v_0 t \cos(\theta)$$